February 04, 2020

5.1 - Day 2 - Factoring Polynomials and Zeros

* OBJ: Students will... Factor polynomials to find the zeros of the function.

Ex1. Write each polynomial in factored form.

a. $x^5 + 13x^4 - 30x^3$

$x^3(x^2+13x-30)$

$x^3(x+15)(x-2)$

b. $4x^3 + 10x^2 - 24x$

$2x(2x^2+5x-12)$

-24, 8, -3, 5

$2x^2+8x-3x-12$

$2x(x+4)-3(x+4)$

$2x(2x-3)(x+4)$

Zeros of Polynomial Functions

If f is a polynomial function, then the values of x for which $f(x)$ is equal to 0 are called the **zeros** of f. These values of x are the **roots**, or **solutions**, of the polynomial equation $f(x) = 0$. Each real root of the polynomial equation appears as an x-intercept of the graph of the polynomial function.

EXAMPLE 5 Finding Zeros of a Polynomial Function

Find all zeros of $f(x) = x^3 + 3x^2 - x - 3$.

$x^2(x+3) - 1(x+3) = 0$

$(x^2-1)(x+3) = 0$

$(x+1)(x-1)(x+3) = 0$

$x = -1, 1, -3$

EXAMPLE 6 Finding Zeros of a Polynomial Function

Find all zeros of $f(x) = -x^4 + 4x^3 - 4x^2$.

$-x^2(x^2-4x+4) = 0$

$-x^2(x-2)(x-2) = 0$

$-x^2 = 0$

$x^2 = 0$

$\sqrt{x^2} = \sqrt{0}$

$x = 0$ (mult. 2)

$x = 2$ (mult. 2)

Multiplicities of Zeros

We can use the results of factoring to express a polynomial as a product of factors. For instance, in Example 6, we can use our factoring to express the function's equation as follows:

$$f(x) = -x^4 + 4x^3 - 4x^2 = -(x^4 - 4x^3 + 4x^2) = -x^2(x - 2)^2.$$

The factor x occurs twice: $x^2 = x \cdot x$.

The factor $(x - 2)$ occurs twice: $(x - 2)^2 = (x - 2)(x - 2)$.

Notice that each factor occurs twice. In factoring the equation for the polynomial function f, if the same factor $x - r$ occurs k times, but not $k + 1$ times, we call r a **zero with multiplicity** k. For the polynomial function

$$f(x) = -x^2(x - 2)^2,$$

0 and 2 are both zeros with multiplicity 2.

EXAMPLE 7 Finding Zeros of a Polynomial Function

Find all zeros of $f(x) = -2x^5 + 20x^4 - 50x^3$

$-2x^3(x^2 - 10x + 25) = 0$

$-2x^3(x-5)(x-5) = 0$

$x = 0$ (mult. 3) $\quad x = 5$ (mult. 2)

Check Point

Find all the zeros of the following polynomial functions.

a. $f(x) = 2x^5 + 10x^4 - 18x^3 - 90x^2$

$2x^2(x^3 + 5x^2 - 9x - 45)$

$x^2(x+5) - 9(x+5)$

$2x^2(x^2 - 9)(x+5)$

$2x^2(x+3)(x-3)(x+5) = 0$

$x = 0$ (mult. 2), $-3, 3, -5$

b. $g(x) = 27x^3 - 75x$

$3x(9x^2 - 25) = 0$

$3x(3x+5)(3x-5) = 0$

$x = 0, -\frac{5}{3}, \frac{5}{3}$

Last Examples:

Write a polynomial function in standard form with the given zeros.

a.) x = 0 (mult. 2) and -3

$x^2(x+3)=0$

$x^3+3x^2=0$

$f(x)=x^3+3x^2$

b.) x = 5 and x = -3/2

$(x-5)(2x+3)=0$

$2x^2-7x-15=0$

$f(x)=2x^2-7x-15$

c.) x = 0 and -4 (mult. 2)

$x(x+4)^2=0$

$x(x+4)(x+4)=0$

$x(x^2+8x+16)=0$

$x^3+8x^2+16x=0$

$f(x)=x^3+8x^2+16x$