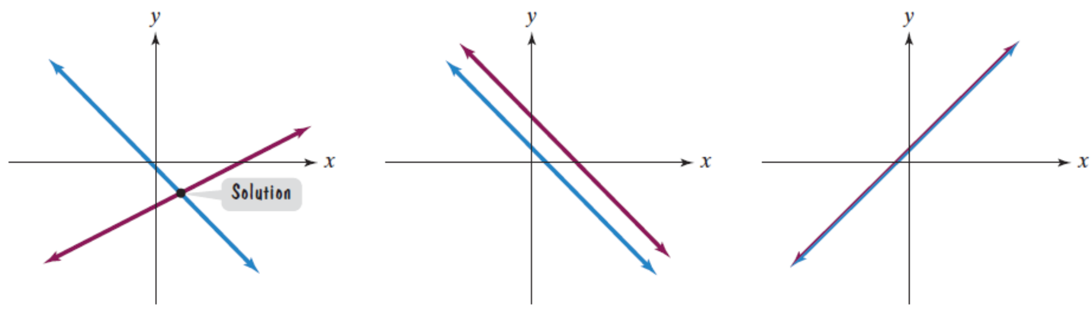
5.1 - Systems of Linear Equations -- Day 2

The Number of Solutions to a System of Two Linear Equations

The number of solutions to a system of two linear equations in two variables is given by one of the following. (See **Figure 7.3**.)

Number of Solutions	What This Means Graphically
Exactly one ordered-pair solution	The two lines intersect at one point.
No solution	The two lines are parallel.
Infinitely many solutions	The two lines are identical.



Exactly one solution | No solution (parallel lines) | Infinitely many solutions (lines coincide)

FIGURE 7.3 Possible graphs for a system of two linear equations in two variables

A linear system with no solution is called an **inconsistent system**. If you attempt to solve such a system by substitution or addition, you will eliminate both variables. A false statement, such as $0 = 12$, will be the result.

EXAMPLE 5

Solve the system:

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$$\begin{cases} 3(4x + 6y = 12) \\ -2(6x + 9y = 12.) \end{cases}$$

$$\boxed{\text{No Solution}}$$

$$\begin{array}{r} \cancel{12x} + \cancel{18y} = 36 \\ + \ -\cancel{12x} - \cancel{18y} = -24 \\ \hline 0 = 12 \end{array}$$

(False) ?

A linear system that has at least one solution is called a **consistent system**. Lines that intersect and lines that coincide both represent consistent systems. If the lines coincide, then the consistent system has infinitely many solutions, represented by every point on either line.

The equations in a linear system with infinitely many solutions are called **dependent**. If you attempt to solve such a system by substitution or addition, you will eliminate both variables. However, a true statement, such as 10 = 10, will be the result.

EXAMPLE 6

Solve the system:

$$\begin{cases} y = 3x - 2 \\ 15x - 5y = 10. \end{cases}$$

$15x - 5(3x - 2) = 10$

$\cancel{15x} - \cancel{15x} + 10 = 10$

$10 = 10$

true $\rightarrow$ $\boxed{\text{Infinite Solutions}}$

Applications

We begin with applications that involve two unknown quantities. We will let x and y represent these quantities. We then model the verbal conditions of the problem with a system of linear equations in x and y.

Strategy for Problem Solving Using Systems of Equations

Step 1 Read the problem carefully. Attempt to state the problem in your own words and state what the problem is looking for. Use variables to represent unknown quantities.

Step 2 Write a system of equations that models the problem's conditions.

Step 3 Solve the system and answer the problem's question.

Step 4 Check the proposed solution in the original wording of the problem.

The sum of two numbers is 7. If one number is subtracted from the other, their difference is -1. Find the numbers.

$$\begin{cases} x+y=7 \\ x-y=-1 \end{cases}$$

$+$

$2x=6$, $\boxed{x=3 \quad 1^{st}\#}$

$3+y=7$

$\boxed{y=4 \quad 2^{nd}\#}$

Three times a first number decreased by a second number is 1. The first number increased by twice the second number is 12. Find the numbers.

$$\begin{cases} 2(3x-y=1) \\ x+2y=12 \end{cases}$$

$\rightarrow$

$$\begin{array}{r} 6x-2y=2 \\ +\quad x+2y=12 \\ \hline 7x=14 \end{array}$$

$x=2$ $1^{st}\#$

$2+2y=12$

$2y=10$

$y=5$ $2^{nd}\#$

A hotel has 200 rooms. Those with kitchen facilities rent for \$100 per night and those without kitchen facilities rent for \$80 per night. On a night when the hotel was completely occupied, revenues were \$17,000. How many of each type of room does the hotel have?

x = rooms w/ Kitchens

y = rooms w/out Kitchens

$$\begin{cases} x+y=200 \longrightarrow x=-y+200 \\ 100x+80y=17000 \end{cases}$$

$100(-y+200)+80y=17000$

$-100y+20000+80y=17000$

$-20y=-3000$

$\boxed{y=150 \text{ rooms w/out Kitchens}}$

$\boxed{x=50 \text{ rooms w/ Kitchens}}$

Paul and Willie each improved their yards by planting hostas and ivy. They bought their supplies from the same store. Paul spent $69 on 5 hostas and 3 pots of ivy. Willie spent $85.20 on 2 hostas and 6 pots of ivy. What is the cost of one hosta and the cost of one pot of ivy?

$$\begin{cases} -2(5x + 3y = 69) \\ 2x + 6y = 85.20 \end{cases}$$

$$+ \quad -10x - 6y = -138$$

$$-8x = -52.80$$

$$\boxed{x = \$6.60}$$

$x =$ cost of one hosta

$y =$ cost of one pot of ivy

$$5(6.60) + 3y = 69$$

$$33 + 3y = 69$$

$$3y = 36$$

$$\boxed{y = \$12.00}$$

hostas

ivy