5.1 - Systems of Linear Equations (More Apps) -- Day 3

Chemists and pharmacists often have to change the concentration of solutions and other mixtures. In these situations, the amount of a particular ingredient in the solution or mixture is expressed as a percentage of the total.

HW Day 3:
#'s: 55, 57, 61, 64
67, 73, 75

EXAMPLE 7 Solving a Mixture Problem

A chemist working on a flu vaccine needs to mix a 10% sodium-iodine solution with a 60% sodium-iodine solution to obtain 50 milliliters of a 30% sodium-iodine solution. How many milliliters of the 10% solution and of the 60% solution should be mixed?

$\begin{cases} x+y=50 \\ .10x+.60y=15 \end{cases}$ $(50 \cdot .30)$

x = mL of 10% solution
y = mL of 60% solution

$x=-y+50$

$.10(-y+50)+.60y=15$

$-.10y+5+.60y=15$

$-5 \quad -5$

$.50y=10$

$.50 \quad .50$

$y=20$ mL of 60% solution

$x=-20+50$

$x=30$ mL of 10% solution

Check Point 7 A chemist needs to mix an 18% acid solution with a 45% acid solution to obtain 12 liters of a 36% acid solution. How many liters of each of the acid solutions must be used?

x y

$(12 \cdot .36)$

$\begin{cases} -.18(x+y=12) \\ .18x+.45y=4.32 \end{cases}$

$+ \quad -.18x-.18y=-2.16$

$.27y=2.16$

$.27 \quad .27$

$y=8$ L of 45% solution

$x+y=12$

$x+8=12$

$x=4$ L of 18% solution

We have seen that if an object moves at an average velocity v, the distance, s, covered in time t is given by the formula

$d = rt$ ← $s = vt$ Distance equals velocity times time.

Recall that objects that move in accordance with this formula are said to be in **uniform motion**. Wind and water current have the effect of increasing or decreasing a traveler's velocity.

EXAMPLE 8 Solving a Uniform Motion Problem

When a small airplane flies with the wind, it can travel 450 miles in 3 hours. When the same airplane flies in the opposite direction against the wind, it takes 5 hours to fly the same distance. Find the average velocity of the plane in still air and the average velocity of the wind.

$450 = V \cdot 3$
$150 = V$

$450 = V \cdot 5$
$90 = V$

x = velocity of plane
y = velocity of wind

$$\begin{cases} x + y = 150 \\ x - y = 90 \end{cases}$$

$2x = 240$, $x = 120$ mph avg. velocity of the plane.

$y = 30$ mph avg. velocity wind

Check Point 8 With the current, a motorboat can travel 84 miles in 2 hours. Against the current, the same trip takes 3 hours. Find the average velocity of the boat in still water and the average velocity of the current.

$84 = V \cdot 3$
$28 = V$

$84 = V \cdot 2$
$42 = V$

$$\begin{cases} x + y = 42 \\ x - y = 28 \end{cases}$$

$2x = 70$

$x = 35$ mph boat

$y = 7$ mph water current

Functions of Business: Break-Even Analysis

Suppose that a company produces and sells x units of a product. Its *revenue* is the money generated by selling x units of the product. Its *cost* is the cost of producing x units of the product.

Revenue and Cost Functions

A company produces and sells x units of a product.

Revenue Function

$$R(x) = (\text{price per unit sold})x$$

Cost Function

$$C(x) = \text{fixed cost} + (\text{cost per unit produced})x$$

The point of intersection of the graphs of the revenue and cost functions is called the **break-even point**. The x-coordinate of the point reveals the number of units that a company must produce and sell so that money coming in, the revenue, is equal to money going out, the cost. The y-coordinate of the break-even point gives the amount of money coming in and going out. Example 9 illustrates the use of the substitution method in determining a company's break-even point.

EXAMPLE 9 Finding a Break-Even Point

Technology is now promising to bring light, fast, and beautiful wheelchairs to millions of people with disabilities. A company is planning to manufacture these radically different wheelchairs. Fixed cost will be \$500,000 and it will cost \$400 to produce each wheelchair. Each wheelchair will be sold for \$600.

a. Write the cost function, C, of producing x wheelchairs.

b. Write the revenue function, R, from the sale of x wheelchairs.

c. Determine the break-even point. Describe what this means.

a.) $C(x) = 500\,000 + 400x$

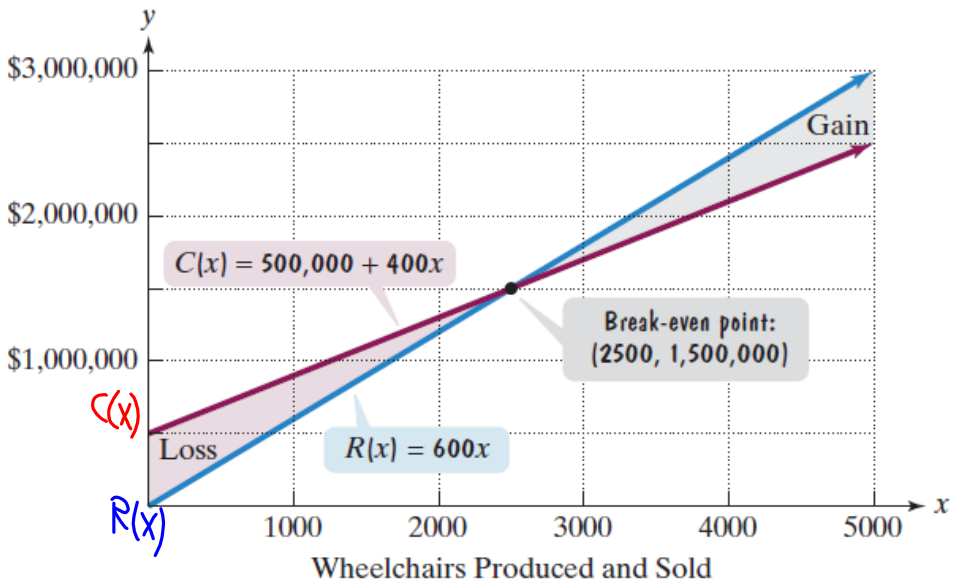
b.) $R(x) = 600x$

c.) $600x = 500\,000 + 400x$

$200x = 500\,000$

$x = 2{,}500$ wheelchairs

$R = C =$ \$1,500,000



The Profit Function

The profit, $P(x)$, generated after producing and selling x units of a product is given by the **profit function**

$$P(x) = R(x) - C(x),$$

where R and C are the revenue and cost functions, respectively.

The profit function for the wheelchair business in Example 7 is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 600x - (500{,}000 + 400x) \\ &= 200x - 500{,}000. \end{aligned}$$