February 11, 2020

5.2 - Solving Polynomials with Real/Non-Real Solutions - Day 1 -

We have used basic factoring to solve polynomial equations already, but all of the zeros (solutions) have been real numbers. This of course will not always be the case. This next section we will cover advanced factoring and solving techniques for solving polynomial equations we have not yet encountered.

Some common factoring we have used in the past.....

GCF (Greatest Common Factor)

ex.) $18x^3 + 12x^2 \dashrightarrow 6x^2(3x + 2)$

Difference of Two Squares

$a^2 - b^2 = (a + b)(a - b)$

ex.) $4x^2 - 49 \dashrightarrow (2x + 7)(2x - 7)$

Advanced Trinomials (a > 1)

ex.) $3x^2 - 2x - 8$

$3x^2 - 6x + 4x - 8$

GCF --> $3x(x - 2) + 4(x - 2)$

$(3x + 4)(x - 2)$

Mult. -24, -6, 4, -2, Sum

Basic Trinomials (a = 1)

ex.) $x^2 + 7x - 18 \dashrightarrow (x + 9)(x - 2)$

Mult. -18, 9, -2, 7, Sum

"Quadratic-Like" Polynomials

Ex.1) Solve for all zeros.

$x^4 - 4x^2 = 45$

$x^4 - 4x^2 - 45 = 0$ (sum, mult)

$(x^2 - 9)(x^2 + 5) = 0$

$(x + 3)(x - 3)(x^2 + 5) = 0$

$x = \pm 3$

$x^2 + 5 = 0$

$\sqrt{x^2} = \sqrt{-5}$

$x = \pm i\sqrt{5}$

Ex.2) Solve for all zeros.

$x^6 - 24x^2 = 2x^4$

$x^6 - 2x^4 - 24x^2 = 0$

$x^2(x^4 - 2x^2 - 24) = 0$ (sum, mult)

$x^2(x^2 - 6)(x^2 + 4) = 0$

$x^2 - 6 = 0$

$\sqrt{x^2} = \sqrt{6}$

$x = \pm\sqrt{6}$

$x^2 + 4 = 0$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

$x = 0$ (mult. 2)

Ex.3) Solve for all zeros.

$$-x^7 + 81x^3 = 0$$

$-x^3(x^4 - 81) = 0$

$-x^3(x^2+9)(x^2-9) = 0$

$-x^3(x^2+9)(x+3)(x-3) = 0$

$x = 0$ (mult. 3)

$x^2+9=0$

$x^2 = -9$

$x = \pm 3i$

$x = \pm 3$

CheckPoint 1) Solve for all zeros.

$$2x^5 - 46x^3 - 100x = 0$$

$2x(x^4 - 23x^2 - 50) = 0$ (sum, mult)

$2x(x^2-25)(x^2+2) = 0$

$2x(x+5)(x-5)(x^2+2) = 0$

$x = 0$

$x = \pm 5$

$x^2+2=0$

$x^2 = -2$

$x = \pm i\sqrt{2}$

Ex.4) Solve for all zeros.

$$x^3 - 2x^2 = -17x$$

$x^3 - 2x^2 + 17x = 0$

$x(x^2 - 2x + 17) = 0$ (sum, mult)

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(17)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{-64}}{2}$

$x = \dfrac{2 \pm 8i}{2}$ simp.

$x = 1 \pm 4i$

$x = 0$