

5.2 - Solving Polynomials with Real/Non-Real Solutions - Day 2 -

Sum and Difference of Cubes

Sum: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Diff: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

*When a and b are perfect cubes.

Perfect Cubes:

- $1^3 \rightarrow 1$
- $2^3 \rightarrow 8$
- $3^3 \rightarrow 27$
- $4^3 \rightarrow 64$
- $5^3 \rightarrow 125$
- $6^3 \rightarrow 216$
- $7^3 \rightarrow 343$
- $8^3 \rightarrow 512$
- $9^3 \rightarrow 729$
- $10^3 \rightarrow 1000$
- ..., ...

Ex.1) Factor the following.

$$x^3 - 216$$

$a = x \quad b = 6$

$(x - 6)(x^2 + 6x + 36)$

$$8x^3 + 27$$

$a = 2x \quad b = 3$

$(2x + 3)(4x^2 - 6x + 9)$

Ex.2) Solve for all zeros.

$$2x^4 = 250x$$

$$2x^4 - 250x = 0$$

$$2x(x^3 - 125) = 0$$

$$2x(x - 5)(x^2 + 5x + 25)$$

$\downarrow \qquad \downarrow$

$$2x = 0$$

$x = 0$

$x = 5$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(25)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{-75}}{2} = \frac{-5 \pm 5i\sqrt{3}}{2}$$

Simp.

$$i\sqrt{75}$$

$$i\sqrt{25 \cdot 3}$$

$$5i\sqrt{3}$$

Checkpoint 1) Solve for all zeros.

$$8x^3 + 27 = 0$$

$$(2x+3)(4x^3 - 6x + 9) = 0$$

$$x = -\frac{3}{2}$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$$

$$x = \frac{6 \pm \sqrt{-108}}{8}$$

$$x = -\frac{3}{2}$$

$$x = \frac{6 \pm 6i\sqrt{3}}{8} \quad \div 2$$

$$x = \frac{3 \pm 3i\sqrt{3}}{4}$$

Checkpoint 2 --- CHALLENGE!!! Solve for all zeros.

$$x^7 - 216x^4 + 3456 = 16x^3$$

$$x^7 - 216x^4 - 16x^3 + 3456 = 0$$

$$x^4(x^3 - 216) - 16(x^3 - 216) = 0$$

$$(x^4 - 16)(x^3 - 216) = 0$$

$$(x^2 + 4)(x^2 - 4)(x - 6)(x^2 + 6x + 36) = 0$$

$$(x^2 + 4) \rightarrow \sqrt{x^2} = \sqrt{-4} \rightarrow x = \pm 2i$$

$$(x^2 - 4) \rightarrow (x+2)(x-2) \rightarrow x = \pm 2$$

$$(x - 6) \rightarrow x = 6$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(36)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{-108}}{2}$$

$$x = \frac{-6 \pm 6i\sqrt{3}}{2}$$

$$x = -3 \pm 3i\sqrt{3}$$