5.2 - Systems of Linear Inequalities and Programming Day 2

The Berlin Airlift (1948–1949) was an operation by the United States and Great Britain in response to military action by the former Soviet Union: Soviet troops closed all roads and rail lines between West Germany and Berlin, cutting off supply routes to the city. The Allies used a mathematical technique developed during World War II to maximize the amount of supplies transported. During the 15-month airlift, 278,228 flights provided basic necessities to blockaded Berlin, saving one of the world's great cities.

In this section, we will look at an important application of systems of linear inequalities. Such systems arise in **linear programming**, a method for solving problems in which a particular quantity that must be maximized or minimized is limited by other factors. Linear programming is one of the most widely used tools in management science. It helps businesses allocate resources to manufacture products in a way that will maximize profit. Linear programming accounts for more than 50% and perhaps as much as 90% of all computing time used for management decisions in business. The Allies used linear programming to save Berlin.

HW Day 2:
p.840:
#'s: 2, 3, 7, 10, 16

Objective Functions in Linear Programming

Many problems involve quantities that must be maximized or minimized. Businesses are interested in maximizing profit. An operation in which bottled water and medical kits are shipped to earthquake survivors needs to maximize the number of survivors helped by this shipment. An **objective function** is an algebraic expression in two or more variables describing a quantity that must be maximized or minimized.

Ex.1 Graph and Solve.

$$\begin{cases} \boxed{1 \le x \le 5} \\ \boxed{y \ge 2} \\ x - y \ge -5 \end{cases}$$

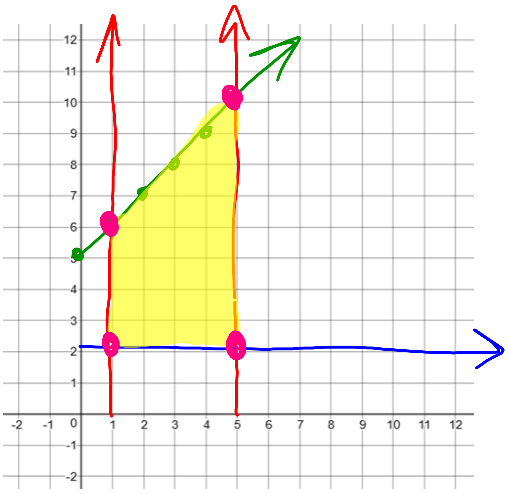
Objective Function:
$z = 3x - 2y$

$-y \ge -x - 5 \;\to\; \boxed{y \le x + 5}$
$-1 \quad -1 \quad -1$

a. *Graph the system of inequalities representing the constraints.*

b. *Find the value of the objective function at each corner of the graphed region.*

c. *Use the values in part (b) to determine the maximum value of the objective function and the values of x and y for which the maximum occurs.*



$(1,6) \to 3(1)-2(6) = -9$

$(1,2) \to 3(1)-2(2) = -1$

$\boxed{(5,2)} \to 3(5)-2(2) = 11$

$(5,10) \to 3(5)-2(10) = -5$

$\boxed{\text{Max at } (5,2) \text{ of } 11}$

Ex.2 Graph and Solve.

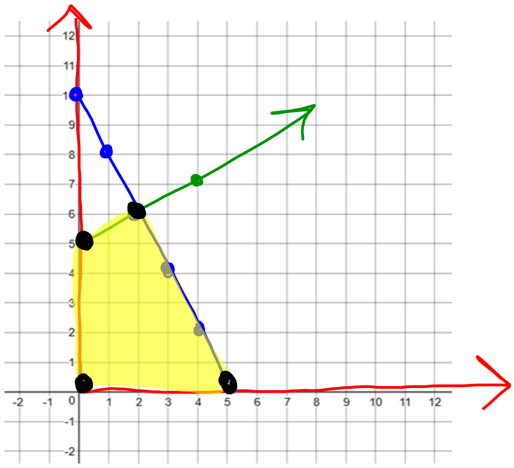
$$\begin{cases} x \geq 0, y \geq 0 \\ 2x + y \leq 10 \\ x - 2y \geq -10 \end{cases}$$

Objective Function:
$z = x + 6y$

a. *Graph the system of inequalities representing the constraints.*
b. *Find the value of the objective function at each corner of the graphed region.*
c. *Use the values in part (b) to determine the maximum value of the objective function and the values of x and y for which the maximum occurs.*

$\frac{-2y}{-2} \geq \frac{-x}{-2} \frac{-10}{-2}$

$\boxed{y \leq \frac{1}{2}x + 5}$



Maximize: $z = x + 6y$

$(0,0)$: $0 + 6(0) = 0$

$(0,5)$: $0 + 6(5) = 30$

$(5,0)$: $5 + 6(0) = 5$

$\boxed{(2,6)}$: $2 + 6(6) = 38$

$\boxed{\text{Max value of 38 at } (2,6).}$

Check Point 1 A company manufactures bookshelves and desks for computers. Let x represent the number of bookshelves manufactured daily and y the number of desks manufactured daily. The company's profits are \$25 per bookshelf and \$55 per desk. Write the objective function that models the company's total daily profit, z, from x bookshelves and y desks. (Check Points 2 through 4 are related to this situation, so keep track of your answers.)

$\boxed{z = 25x + 55y}$

Profit (\$)

Constraints in Linear Programming

Ideally, the number of earthquake survivors helped in Example 1 should increase without restriction so that every survivor receives water and medical supplies. However, the planes that ship these supplies are subject to weight and volume restrictions. In linear programming problems, such restrictions are called **constraints**. Each constraint is expressed as a linear inequality. The list of constraints forms a system of linear inequalities.

Check Point 2 To maintain high quality, the company in Check Point 1 should not manufacture more than a total of 80 bookshelves and desks per day. Write an inequality that models this constraint.

$x+y \leq 80$

Check Point 3 To meet customer demand, the company in Check Point 1 must manufacture between 30 and 80 bookshelves per day, inclusive. Furthermore, the company must manufacture at least 10 and no more than 30 desks per day. Write an inequality that models each of these sentences. Then summarize what you have described about this company by writing the objective function for its profits and the three constraints.

$30 \leq x \leq 80$

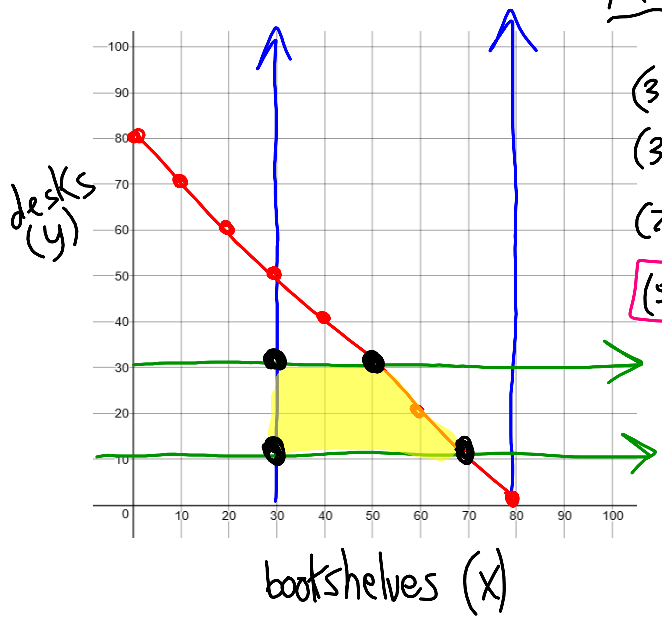
$10 \leq y \leq 30$

Solving a Linear Programming Problem

Let $z = ax + by$ be an objective function that depends on x and y. Furthermore, z is subject to a number of constraints on x and y. If a maximum or minimum value of z exists, it can be determined as follows:

1. Graph the system of inequalities representing the constraints.
2. Find the value of the objective function at each corner, or **vertex**, of the graphed region. The maximum and minimum of the objective function occur at one or more of the corner points.

Check Point 4 For the company in Check Points 1–3, how many bookshelves and how many desks should be manufactured per day to obtain maximum profit? What is the maximum daily profit?



Maximize: $z = 25x + 55y$

$(30,10) \longrightarrow \$1300$

$(30,30) \longrightarrow \$2400$

$(70,10) \longrightarrow \$2300$

$\boxed{(50,30)} \longrightarrow \boxed{\$2900}$