

## 5.2 - Day 3 - Dividing Polynomials

Use synthetic division to divide  $x^3 - 14x^2 + 51x - 54$  by  $x + 2$ . What is the quotient and remainder?

**Step 1** Reverse the sign of +2. Write the coefficients of the polynomial.

$$\underline{-2} \mid 1 \quad -14 \quad 51 \quad -54$$

**Step 3** Multiply the coefficient by the divisor. Add to the next coefficient.

$$\begin{array}{r|rrrr} -2 & 1 & -14 & 51 & -54 \\ & & -2 & & \\ \hline & 1 & -16 & & \end{array}$$

**Step 2** Bring down the first coefficient.

$$\begin{array}{r|rrrr} -2 & 1 & -14 & 51 & -54 \\ & & & & \\ \hline & 1 & & & \end{array}$$

**Step 4** Continue multiplying and adding through the last coefficient.

$$\begin{array}{r|rrrr} -2 & 1 & -14 & 51 & -54 \\ & & -2 & 32 & -166 \\ \hline & 1 & -16 & 83 & -220 \end{array}$$

The quotient is  $x^2 - 16x + 83$ , R  $-220$ .

### Ex. 1

Use synthetic division to divide  $(x^3 + 5x^2 - x - 9)$  by  $(x + 2)$ .

$$\begin{array}{r|rrrr} -2 & 1 & 5 & -1 & -9 \\ & & -2 & -6 & 14 \\ \hline & 1 & 3 & -7 & 5 \end{array}$$

$\downarrow$     $\downarrow$     $\downarrow$     $\downarrow$

$x^2 + 3x - 7, R 5$

remainder

### The Factor Theorem

Let  $f(x)$  be a polynomial.

- If  $f(c) = 0$ , then  $x - c$  is a factor of  $f(x)$ .
- If  $x - c$  is a factor of  $f(x)$ , then  $f(c) = 0$ .

### Ex. 2

Use synthetic division to determine if the given binomial is a factor of the polynomial:

$$(x^3 - 44x + 48) ; (x - 6)$$

$$\begin{array}{r|rrrr} 6 & 1 & 0 & -44 & 48 \\ & & 6 & 36 & -48 \\ \hline & 1 & 6 & -8 & 0 \end{array}$$

Yes,  $(x-6)$  is a factor.

0 remainder

### Ex. 3

Use synthetic division to divide  $(x^3 - 5x^2 - 9x + 45)$  by  $(x - 5)$ . Then factor the quotient completely.

$$\begin{array}{r|rrrr} 5 & 1 & -5 & -9 & 45 \\ & & 5 & 0 & -45 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

Factor.....

$$(x+3)(x-3)(x-5)$$

$x^2 - 9$

**Ex. 4**

Use synthetic division to divide  $(x^3 - 57x + 56)$  by  $(x - 7)$ . Then factor the quotient completely.

$$\begin{array}{r|rrrr} 7 & 1 & 0 & -57 & 56 \\ & & 7 & 49 & -36 \\ \hline & 1 & 7 & -8 & 0 \end{array}$$

$$x^2 + 7x - 8$$

$$(x+8)(x-1)(x-7)$$

8	-8
7	-1

**The Remainder Theorem**

If the polynomial  $f(x)$  is divided by  $x - c$ , then the remainder is  $f(c)$ .

*... allows us to use synthetic division to evaluate a polynomial...*

**Ex.5** Evaluate  $f(x) = x^5 - 2x^3 - x^2 + 2$ , at  $x = 3$ .

$$\begin{array}{r|rrrrrr} 3 & 1 & 0 & -2 & -1 & 0 & 2 \\ & & 3 & 9 & 21 & 60 & 180 \\ \hline & 1 & 3 & 7 & 20 & 60 & 182 \end{array}$$

answer!

$(x-3)$  is NOT a factor!