5.2 Day 4 - Solving Polynomial Equations w/ Synthetic Division

The Rational Zero Theorem

If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has *integer* coefficients and $\frac{p}{q}$ (where $\frac{p}{q}$ is reduced to lowest terms) is a rational zero of f, then p is a factor of the constant term, a_0, and q is a factor of the leading coefficient, a_n.

$$\boxed{\text{Possible rational zeros} = \frac{\text{Factors of the constant term}}{\text{Factors of the leading coefficient}}}$$

EXAMPLE 1 Using the Rational Zero Theorem

List all possible rational zeros of $f(x) = -x^4 + 3x^2 + 4.$

$$\frac{\pm1, \pm2, \pm4}{\pm1} = \boxed{\pm1, \pm2, \pm4}$$

EXAMPLE 2 Using the Rational Zero Theorem

List all possible rational zeros of $f(x) = 15x^3 + 14x^2 - 3x - 2.$

$$\frac{\pm1, \pm2}{\pm1, \pm3, \pm5, \pm15} = \boxed{\pm1, \pm\frac{1}{3}, \pm\frac{1}{5}, \pm\frac{1}{15}, \pm2, \pm\frac{2}{3}, \pm\frac{2}{5}, \pm\frac{2}{15}}$$

Using the Rational Zero Theorem to Solve:

1. List all possible rational roots.
2. Test the roots using synthetic division until you get a remainder of zero.
3. Drop down the new coefficients and solve the remaining polynomial for x, using factoring methods or quadratic formula.

EXAMPLE 3 Finding Zeros of a Polynomial Function

Find all zeros of $f(x) = x^3 + 2x^2 - 5x - 6.$

1.) $\frac{\pm1, \pm2, \pm3, \pm6}{\pm1} = \pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -5 & -6 \\ & & 1 & 3 & -2 \\ \hline & 1 & 3 & -2 & -8 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$\boxed{x=-1}$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$\boxed{x=-3}$ $\boxed{x=2}$

EXAMPLE 4 Finding Zeros of a Polynomial Function

Find all zeros of $f(x) = x^3 + 7x^2 + 11x - 3$.

$$\frac{\pm 1, \pm 3}{\pm 1} = \pm 1, \pm 3$$

$$\begin{array}{r|rrrr} 1 & 1 & 7 & 11 & -3 \\ & & 1 & 8 & 19 \\ \hline & 1 & 8 & 19 & 16 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & 7 & 11 & -3 \\ & & 3 & 30 & 123 \\ \hline & 1 & 10 & 41 & 120 \end{array}$$

$\boxed{x=-3}$

$$\begin{array}{r|rrrr} -1 & 1 & 7 & 11 & -3 \\ & & -1 & -6 & -5 \\ \hline & 1 & 6 & 5 & -8 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 7 & 11 & -3 \\ & & -3 & -12 & 3 \\ \hline & 1 & 4 & -1 & 0 \end{array}$$

$$x^2 + 4x - 1 = 0 \quad a \; b \; c$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{20}}{2} \rightsquigarrow \frac{-4 \pm 2\sqrt{5}}{2} = \boxed{x = -2 \pm \sqrt{5}}$$

$\boxed{x=-3}$

Check Point

Find all zeros of $2x^3 + x^2 - 7x - 6 = 0$

$$\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$$

$$\begin{array}{r|rrrr} 1 & 2 & 1 & -7 & -6 \\ & & 2 & 3 & -4 \\ \hline & 2 & 3 & -4 & -10 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 2 & 1 & -7 & -6 \\ & & -2 & 1 & 6 \\ \hline & 2 & -1 & -6 & 0 \end{array}$$

$\boxed{x=-1}$

$$2x^2 - x - 6 = 0$$

-12, -4, 3, -1

$$2x^2 - 4x + 3x - 6 = 0$$

$$2x(x-2) + 3(x-2) = 0$$

$$(2x+3)(x-2) = 0$$

$\boxed{x = -\frac{3}{2}}$ $\boxed{x=2}$

$\boxed{x=-1}$