5.2 Day 5 - Writing Polynomial Functions Given the Roots

take note

Theorem Conjugate Root Theorem

If $P(x)$ is a polynomial with *rational* coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

If $P(x)$ is a polynomial with *real* coefficients, then the complex roots of $P(x) = 0$ occur in conjugate pairs. That is, if $a + bi$ is a complex root with a and b real, then $a - bi$ is also a root.

Irrational and complex roots come in $\pm$ pairs!
(square roots) *(i's)*

Ex.1

A quartic polynomial $P(x)$ has rational coefficients. If $\sqrt{2}$ and $1 + i$ are roots of $P(x) = 0$, what are the two other roots?

$-\sqrt{2}$ $1-i$

A cubic polynomial $P(x)$ has real coefficients. If $3 - 2i$ and $\frac{5}{2}$ are two roots of $P(x) = 0$, what is one additional root?

$3+2i$

Ex.2

Write a polynomial function with rational coefficients so that $P(x) = 0$ has the roots -5 and $2i$.

$x^2 = (\pm 2i)^2$

$x^2 = 4i^2$

$x^2 = -4$

$x^2 + 4 = 0$

$* \ i^2 = -1$

$x = -5$

$x + 5 = 0$

$(x+5)(x^2+4)$

$$\boxed{x^3 + 5x^2 + 4x + 20 = 0}$$

Ex.3

Write a polynomial function with rational coefficients so that P(x) = 0

has roots of 0(*mult.3*), 2, and $\sqrt{5}$.

$x^2 = \pm\sqrt{5}^2$

$x^2 = 5$

$x^2 - 5 = 0$

$(x^2 - 5)$

$x^3 = 0$ $x = 2$

x^3 $x - 2 = 0$

$(x-2)$

$x^3\underbrace{(x-2)(x^2-5)}$

$x^3(x^3 - 2x^2 - 5x + 10)$

$$\boxed{x^6 - 2x^5 - 5x^4 + 10x^3 = 0}$$

Ex. 4

Write a polynomial function with rational coefficients so that P(x) = 0

has the roots $\frac{1}{4}$ and $3+2i$

$x = \frac{1}{4}$

$4x = 1$

$4x - 1 = 0$

$(4x-1)$ $\cdot$

$x = 3 \pm 2i$

$-3 \quad -3$

$(x-3)^2 = (\pm 2i)^2$

$x^2 - 6x + 9 = 4i^2$

$x^2 - 6x + 9 = -4$

$(x^2 - 6x + 13) = 0$

$4x^3 - 24x^2 + 52x - x^2 + 6x - 13 = 0$

$$\boxed{4x^3 - 25x^2 + 58x - 13 = 0}$$