Topic #7: Graphs of Rational Functions

7.3 - Day 1 - Properties of Rational Function Graphs

Rational functions are quotients of polynomial functions. This means that rational functions can be expressed as

$$f(x) = \frac{p(x)}{q(x)},$$

Day 1 --- HW p.521 #'s: 1 - 4 (*find domain and x-y intercepts*), 17 - 28 all

where p and q are polynomial functions and $q(x) \neq 0$. The **domain** of a rational function is the set of all real numbers except the x-values that make the denominator zero. For example, the domain of the rational function

$$f(x) = \frac{x^2 + 7x + 9}{x(x-2)(x+5)}$$

This is $p(x)$.
This is $q(x)$.

is the set of all real numbers except 0, 2, and −5.

Ex1. Find the domain (restricted values) of each rational function.

$$f(x) = \frac{7x}{-x^2 - 2x + 15}$$

Domain: $x \neq -5, 3$

$$\frac{7x}{-1(x^2+2x-15)}$$

$$\frac{7x}{-1(x+5)(x-3)}$$

$$f(x) = \frac{9x^2 - 4}{12x^2 + 8x}$$

$$\frac{(3x+2)(3x-2)}{4x(3x+2)}$$

Domain: $x \neq -\frac{2}{3}, 0$

Ex2. Find the x and y-intercepts of each rational function.

How to......

- **Find the *y*-intercept** (*if there is one*) by evaluating the function at $f(0)$. *(Plug in zero for x, and calculate.)*
- **Find the x-intercept(s)** (*if there is any*) by setting the numerator equal to zero and solving for x.

$$f(x) = \frac{3x - 3}{x^2 - 1}$$

- y-int: $\frac{3(0)-3}{(0)^2-1} = 3$

$(0,3)$

- x-int: $3x-3=0$ Solve.

$3x = 3$

$x = 1$

$(1,0)$

$$f(x) = \frac{x^2 + 2x}{x^2 + 3}$$

- y-int: $\frac{(0)^2+2(0)}{(0)^2+3} = \frac{0}{3} = 0$

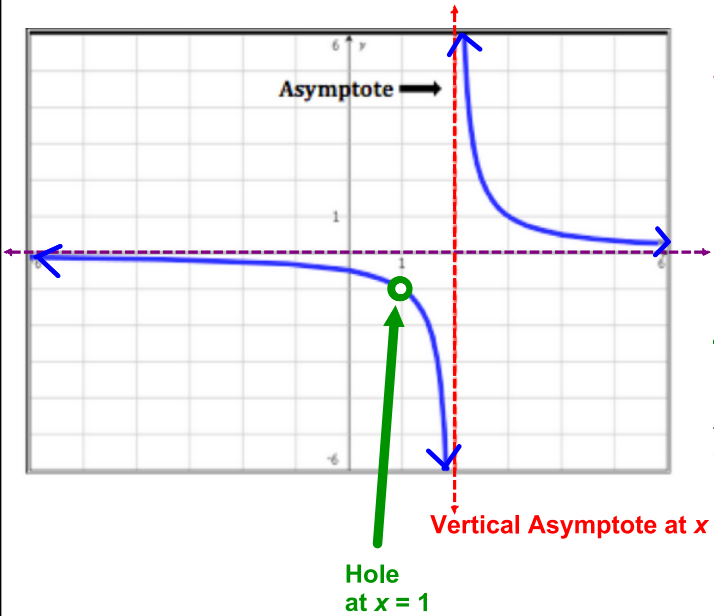
$(0,0)$

- x-int: $x^2+2x=0$

$x(x+2)=0$

$x=0, x=-2$

$(0,0)$
$(-2,0)$



Asymptote: A line that a graph of a function approaches but *never crosses.

Point of Discontinuity (**Hole**): A "hole" in a graph. A single point, or value of x, that is restricted.

How to find the vertical asymptotes and holes of a rational function:

1. Factor everything first.
2. The zeros of the factors of x that **cancel** from the denominator represent a point of discontinuity **(hole)**.
3. The zeros of the factors of x that are **left over** in the denominator represent the **vertical asymptote(s) (VA)**.

Ex3.

$$f(x)=\frac{5}{x^2+6x} \qquad \frac{5}{x(x+6)}$$

$\boxed{\text{Holes: none}}$

$\boxed{\text{VA: at } x=0,\ x=-6}$

$$f(x)=\frac{x-3}{x^2-9} \qquad \frac{\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}$$

$\boxed{\text{Hole: at } x=3}$

$\boxed{\text{VA: at } x=-3}$

$$f(x)=\frac{x^2+x-12}{x^2-3x} \qquad \frac{(x+4)\cancel{(x-3)}}{x\cancel{(x-3)}}$$

$\boxed{\text{Hole at } x=3}$

$\boxed{\text{VA at } x=0}$

Finding Horizontal asymptotes:

Lets let, **n** = degree of numerator
d = degree of denominator

1. If **n<d**; the horizontal asymptote is **y = 0.** ex. $\frac{x}{x^2 + 3}$

2. If **n>d**; there is **NO** horizontal asymptote. ex. $\frac{x^2+5}{4x}$

3. If **n=d**; the horizontal asymptote is **y = a/b** ex. $\frac{3x^1 + 1}{1x^1 - 2}$ HA at $y = 3$

*Where **a** and **b** are the leading coefficients of the numerator and denominator.*

Ex4.

What is the horizontal asymptote for the rational function?

a. $y = \frac{-2x^1 + 6}{x^1 - 5}$ b. $y = \frac{x - 1}{x^2 + 4x + 4}$ c. $y = \frac{x^2 + 2x - 3}{x - 2}$

$\boxed{\text{HA: at } y=-2}$ $\boxed{\text{HA: at } y=0}$ $\boxed{\text{HA: none}}$

Ex5.

Lets put it all together!

Find the horizontal asymptote (HA), holes, and vertical asymptotes (VA) if they exist for each function. Also, find the x and y intercepts.

$$f(x) = \frac{2x^2 - 5x - 12}{x^2 - 3x - 4}$$

$\boxed{\text{HA: at } y=2}$ $\boxed{\text{y-int: } (0,3)}$

X-int: $2x^2 - 5x - 12 = 0$

$(2x+3)(x-4) = 0$

$x = -\frac{3}{2}$ $x = 4$

$\left(-\frac{3}{2}, 0\right)$

$(4, 0)$

$$\frac{(2x+3)\cancel{(x-4)}}{\cancel{(x-4)}(x+1)}$$

$\boxed{\text{VA: at } x=-1}$

$\boxed{\text{Hole: at } x=4}$