9.1 - Right Triangle Trig. - Day 1

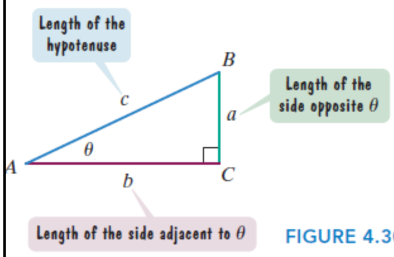


FIGURE 4.30

In solving certain kinds of problems, it is helpful to interpret trigonometric functions in right triangles, where angles are limited to acute angles. **Figure 4.30** shows a right triangle with one of its acute angles labeled θ. The side opposite the right angle, the hypotenuse, has length c. The other sides of the triangle are described by their position relative to the acute angle θ. One side is opposite θ. The length of this side is a. One side is adjacent to θ. The length of this side is b.

Right Triangle Definitions of Trigonometric Functions

See **Figure 4.30**. The six **trigonometric functions of the acute angle θ** are defined as follows:

$$\sin\theta = \frac{\text{length of side opposite angle } \theta}{\text{length of hypotenuse}} = \frac{a}{c} \qquad \csc\theta = \frac{\text{length of hypotenuse}}{\text{length of side opposite angle } \theta} = \frac{c}{a}$$

$$\cos\theta = \frac{\text{length of side adjacent to angle } \theta}{\text{length of hypotenuse}} = \frac{b}{c} \qquad \sec\theta = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to angle } \theta} = \frac{c}{b}$$

$$\tan\theta = \frac{\text{length of side opposite angle } \theta}{\text{length of side adjacent to angle } \theta} = \frac{a}{b} \qquad \cot\theta = \frac{\text{length of side adjacent to angle } \theta}{\text{length of side opposite angle } \theta} = \frac{b}{a}$$

Remember this? → SOHCAHTOA (pronounced: so-cah-tow-ah)

may be helpful in remembering the definitions for sine, cosine, and tangent.

$$\underset{\text{Sine}}{\overset{\uparrow}{S}}\ \underbrace{OH}_{\frac{\text{opp}}{\text{hyp}}}\quad \underset{\text{Cosine}}{\overset{\uparrow}{C}}\ \underbrace{AH}_{\frac{\text{adj}}{\text{hyp}}}\quad \underset{\text{Tangent}}{\overset{\uparrow}{T}}\ \underbrace{OA}_{\frac{\text{opp}}{\text{adj}}}$$

EXAMPLE 1 Evaluating Trigonometric Functions

Find the value of each of the six trigonometric functions of θ in **Figure 4.32**.

Pythag. Thrm. $a^2+b^2=c^2$

$5^2+12^2=c^2$ $\quad c=13$

$169=c^2$

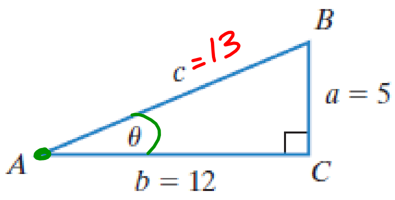


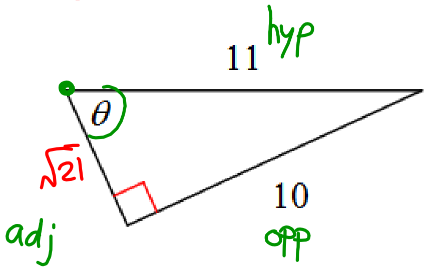
FIGURE 4.32

$\sin A = \boxed{\frac{5}{13}}$ $\quad \csc A = \boxed{\frac{13}{5}}$

$\cos A = \boxed{\frac{12}{13}}$ $\quad \sec A = \boxed{\frac{13}{12}}$

$\tan A = \boxed{\frac{5}{12}}$ $\quad \cot A = \boxed{\frac{12}{5}}$

Example 2 Find the value of each of the six trigonometric functions of θ



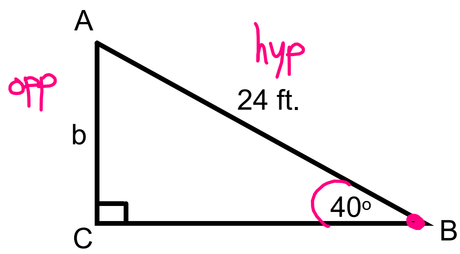
$$a^2+10^2=11^2$$
$$a^2+100=121$$
$$a^2=21$$
$$a=\sqrt{21}$$

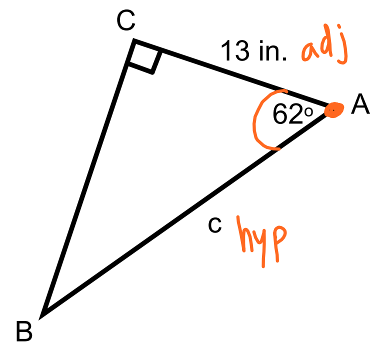
$\sin\theta = \boxed{\frac{10}{11}}$ $\quad \csc\theta = \boxed{\frac{11}{10}}$

$\cos\theta = \boxed{\frac{\sqrt{21}}{11}}$ $\quad$ * $\sec\theta = \frac{11}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \boxed{\frac{11\sqrt{21}}{21}}$

* $\tan\theta = \frac{10}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \boxed{\frac{10\sqrt{21}}{21}}$ $\quad \cot\theta = \boxed{\frac{\sqrt{21}}{10}}$

Example 3 Find the measure of the **side** of the right triangles whose length is designated by a lowercase letter. *Round answers to the nearest tenth.*

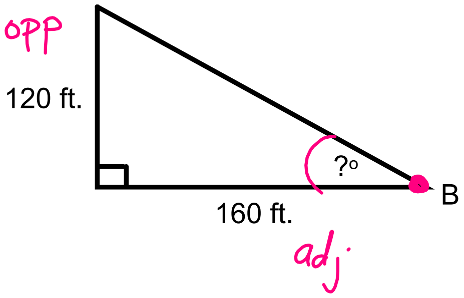


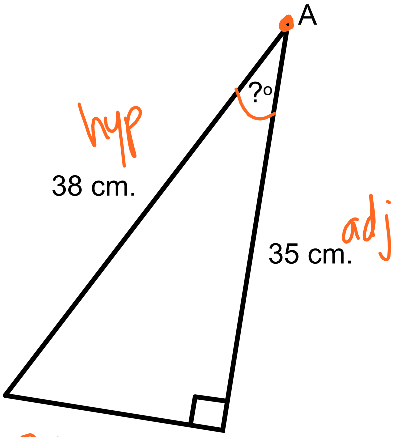


$$24 \cdot \sin(40) = \frac{b}{24} \cdot 24$$
$$\boxed{b = 15.4 \text{ ft.}}$$

$$c \cdot \cos(62) = \frac{13}{c} \cdot c$$
$$\frac{c \cdot \cos(62)}{\cos(62)} = \frac{13}{\cos(62)}$$
$$\boxed{c = 27.7 \text{ in.}}$$

Example 4 Find the measure of the **angle** of the right triangles whose degree is designated by an uppercase letter. *Round answers to the nearest tenth.*





$$\tan B = \frac{120}{160}$$

$$\tan^{-1}\left(\frac{120}{160}\right) = B$$

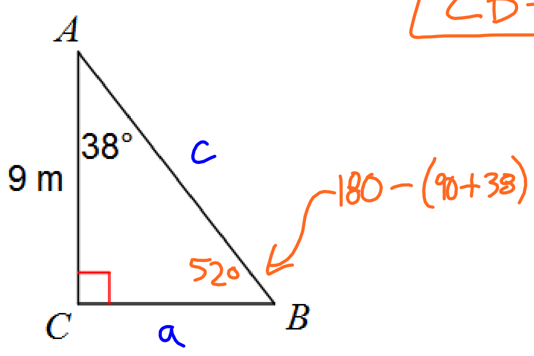
$$\boxed{\angle B = 36.9°}$$

$$\cos A = \frac{35}{38}$$

$$\angle A = \cos^{-1}\left(\frac{35}{38}\right) = \boxed{22.9°}$$

Example 5 **Solve the entire triangle.** (Find each missing side and angle) *Round answers to the nearest tenth.*

$$\boxed{\angle B = 52°}$$



$$\tan(38) = \frac{a}{9}$$

$$\boxed{a = 7.0 \text{ m}}$$

$$\cos(38) = \frac{9}{c}$$

$$\boxed{c = 11.4 \text{ m}}$$