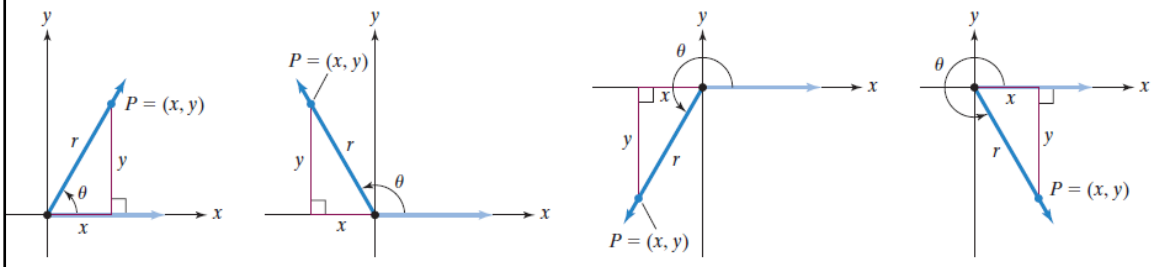
9.2 - Day 2 - Angles in Any Quadrants



(a) θ lies in quadrant I. (b) θ lies in quadrant II. (c) θ lies in quadrant III. (d) θ lies in quadrant IV.

Definitions of Trigonometric Functions of Any Angle

Let θ be any angle in standard position and let $P = (x, y)$ be a point on the terminal side of θ. If $r = \sqrt{x^2 + y^2}$ is the distance from $(0, 0)$ to (x, y), as shown in **Figure 4.41** on the previous page, the **six trigonometric functions of θ** are defined by the following ratios:

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y},\ y \neq 0$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x},\ x \neq 0$$

$$\tan\theta = \frac{y}{x},\ x \neq 0 \qquad \cot\theta = \frac{x}{y},\ y \neq 0.$$

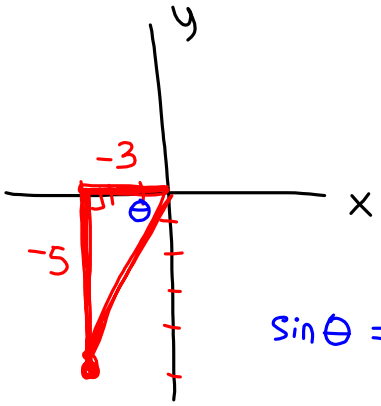
The ratios in the second column are the reciprocals of the corresponding ratios in the first column.

HW 9.2 --- Day 2:

#'s: 1 - 7 odds, 17 - 22 all

EXAMPLE 1 Evaluating Trigonometric Functions

Let $P = (-3, -5)$ be a point on the terminal side of θ. Find each of the six trigonometric functions of θ.



$x^2 + y^2 = r^2$

$(-3)^2 + (-5)^2 = r^2$

$34 = r^2$

$\sqrt{34} = r,\ x = -3,\ y = -5$

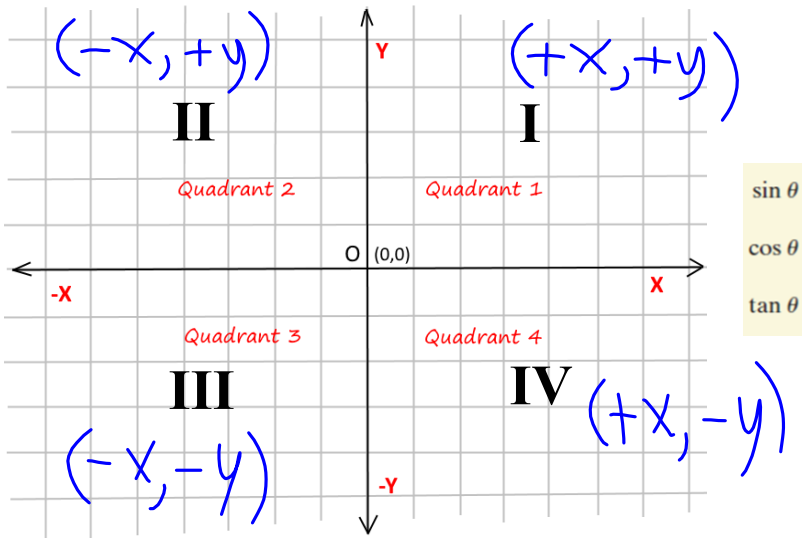
$\sin\theta = -\frac{5}{\sqrt{34}} \rightsquigarrow \boxed{-\frac{5\sqrt{34}}{34}}$ $\csc\theta = \boxed{-\frac{\sqrt{34}}{5}}$

$\cos\theta = -\frac{3}{\sqrt{34}} \rightsquigarrow \boxed{-\frac{3\sqrt{34}}{34}}$ $\sec\theta = \boxed{-\frac{\sqrt{34}}{3}}$

$\tan\theta = \boxed{\frac{5}{3}}$ $\cot\theta = \boxed{\frac{3}{5}}$

Check Point 1 Let $P = (1, -3)$ be a point on the terminal side of θ. Find each of the six trigonometric functions of θ.

Let's Talk About Quadrants and Signs!



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}, y \neq 0$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}, x \neq 0$$

$$\tan\theta = \frac{y}{x}, x \neq 0 \qquad \cot\theta = \frac{x}{y}, y \neq 0.$$

EXAMPLE 3 Finding the Quadrant in Which an Angle Lies

If $\sin\theta > 0$ and $\cos\theta < 0$, name the quadrant in which angle θ terminates in.

$+\frac{y}{r}$ $\quad -\frac{x}{r}$

I or II

If $\sin\theta < 0$ and $\cos\theta > 0$, name the quadrant in which angle θ terminates in.

$-y$ $\quad +x$

III or IV

If $\tan\theta > 0$ and $\sin\theta < 0$, name the quadrant in which angle θ terminates in.

$\frac{+y}{+x}$ OR $\frac{-y}{-x}$ $\quad -y$

I or III

If $\cot\theta < 0$ and $\cos\theta > 0$, name the quadrant in which angle θ terminates in.

$\frac{-x}{+y}$ OR $\frac{+x}{-y}$ $\quad +x$

II or IV

EXAMPLE 3 Finding the Quadrant in Which an Angle Lies

If $\csc\theta < 0$ and $\cos\theta < 0$, name the quadrant in which angle θ terminates in.

$\frac{r}{-y}$ $\quad -x$

III or IV

If $\cot\theta < 0$ and $\csc\theta > 0$, name the quadrant in which angle θ terminates in.

$\frac{-x}{y}$ OR $\frac{x}{-y}$ $\quad \frac{r}{+y}$

II or IV

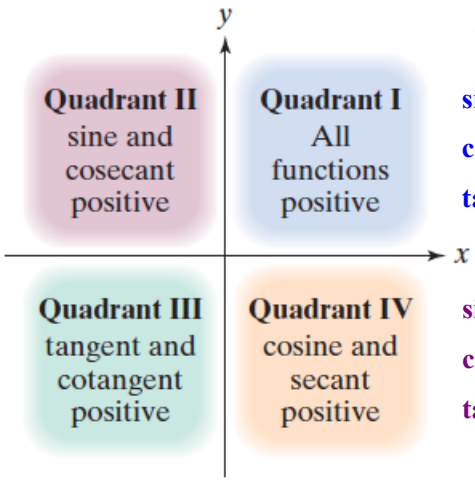
NICE SUMMARY!

$(-x, +y)$

sin and csc +

cos and sec -

tan and cot -



$(+x, +y)$

sin and csc +

cos and sec +

tan and cot +

sin and csc -

cos and sec -

tan and cot +

$(-x, -y)$

sin and csc -

cos and sec +

tan and cot -

$(+x, -y)$

FIGURE 4.47 The signs of the trigonometric functions