

9.2 - Day 5 - Trig. Functions of Any Angle

The Unit Circle

The **unit circle** is a circle of radius 1, with its center at the origin of a rectangular coordinate system. The equation of this unit circle is $x^2 + y^2 = 1$. **Figure 4.19** shows a unit circle with a central angle measuring t radians.

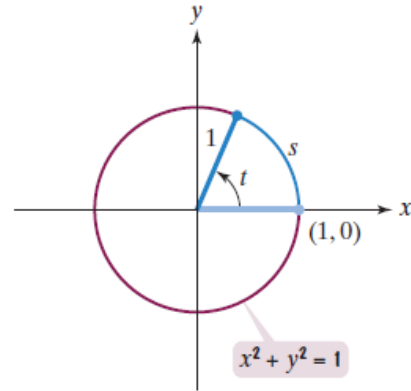
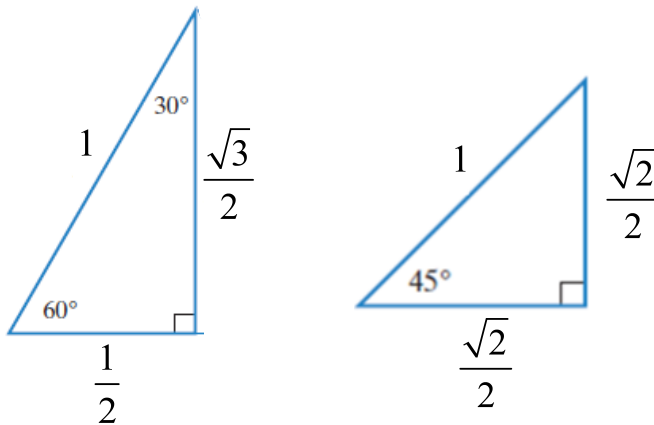


FIGURE 4.19 Unit circle with a central angle measuring t radians

Recall: Special Right Triangles if Hypot. = 1



HW 9.2 Day 5 -
#s: 9 - 15 odds,
61 - 85 odds

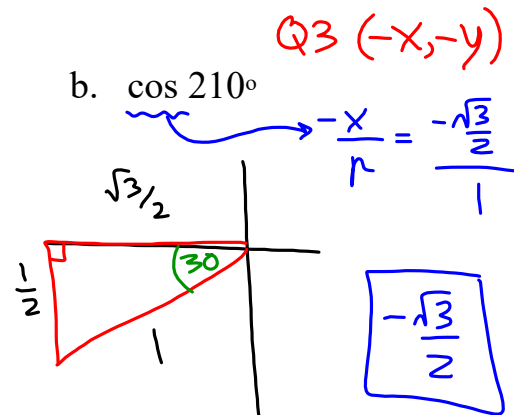
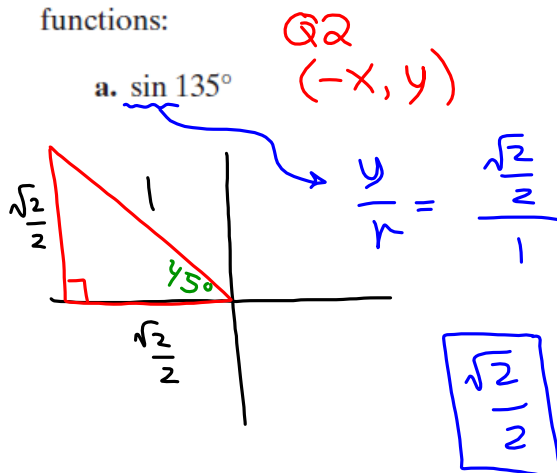
A Procedure for Using Reference Angles to Evaluate Trigonometric Functions

The value of a trigonometric function of any angle θ is found as follows:

1. Find the associated reference angle, θ' , and the function value for θ' .
2. Use the quadrant in which θ lies to prefix the appropriate sign to the function value in step 1.

EXAMPLE 7 Using Reference Angles to Evaluate Trigonometric Functions

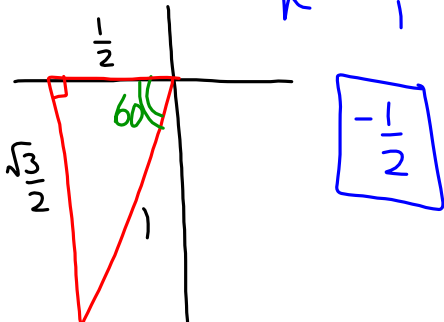
Use reference angles to find the exact value of each of the following trigonometric functions:



Use reference angles to find the exact value of

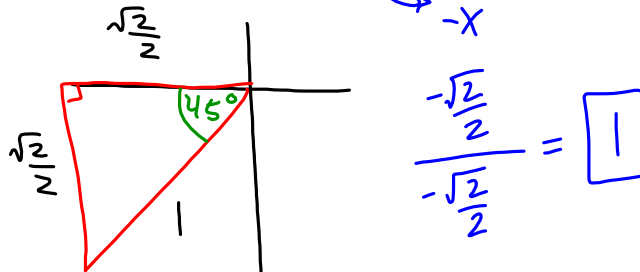
EXAMPLE Q3 (-x, -y)

b. $\cos \frac{4\pi}{3}$ (240°)
 $\rightarrow -x = -\frac{1}{2}$



Check Point Q3 (-x, -y)

b. $\tan \frac{5\pi}{4}$ (225°)
 $\rightarrow -y = -\frac{1}{x}$

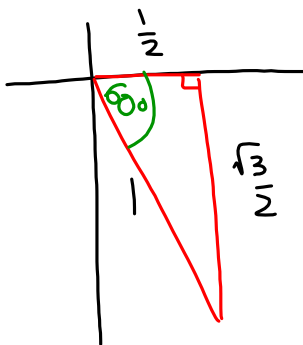


Use reference angles to find the exact value of

EXAMPLE Q4 (x, -y)

Check Point

c. $\cot\left(-\frac{\pi}{3}\right)$ (-60°)
 $\rightarrow -\frac{x}{y} = \frac{1/2}{-\frac{\sqrt{3}}{2}}$



c. $\sec\left(-\frac{\pi}{6}\right)$

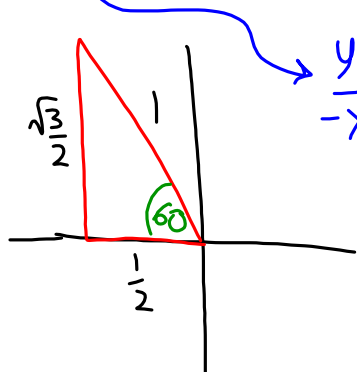
$$\frac{1}{2} \cdot -\frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$$

Use reference angles to find the exact value of

EXAMPLE Q2 (-x, y) (120°)

✓ Check Point Q4 (x, -y)

a. $\tan \frac{14\pi}{3} - \frac{6\pi}{3} - \frac{6\pi}{3} = \frac{2\pi}{3}$



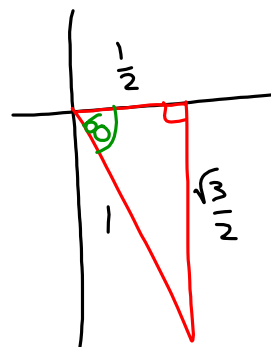
$$\frac{y}{-x} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$

$$\frac{\sqrt{3}/2}{-1/2} = \boxed{-\sqrt{3}}$$

b. $\sec 660^\circ - 360 = 300^\circ$

$$\frac{1}{x} = \frac{1}{-1/2}$$

$$\boxed{2}$$



Trigonometric Functions of Quadrantal Angles

Find the exact value of the following:

a.) $\cos \pi$

$(-1, 0)$
x y

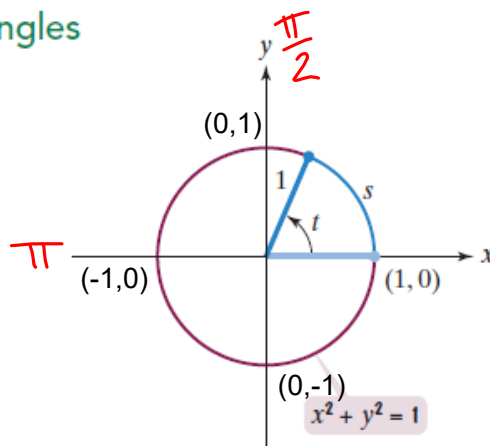
$$\frac{y}{x} = \frac{0}{-1} = \boxed{-1}$$

b.) $\tan \frac{\pi}{2}$

$(0, 1)$

$$\frac{y}{x} = \frac{1}{0}$$

$\boxed{\text{undefined}}$



Trigonometric Functions of Quadrantal Angles

Find the exact value of the following:

c.) $\csc 2\pi$

$(1, 0)$
x y
 $\frac{x}{y} = \frac{1}{0}$

undef.

d.) $\cot \frac{3\pi}{2}$

$(0, -1)$
x y
 $\frac{x}{y} = \frac{0}{-1} = 0$

