D.5 - Exponential/Logarithmic Modeling and Applications

Day 2

Advanced Compound Interest Model:

$$y = P\left(1 + \frac{r}{n}\right)^{nt}$$

y = the amount of money after time has passed

P = the principal (*initial*) amount invested

r = annual rate of interest (*as a decimal*)

t = number of years

n = number of times interest is compounded per year

Mar 14-7:18 AM

Ex.1 An amount of $1,500.00 is deposited in a bank paying an annual interest rate of 4.5%, compounded quarterly. What is the balance after 6 years?

$y = 1500\left(1 + \left(\frac{.045}{4}\right)\right)^{4 \cdot 6}$

$y \approx \$1961.99$

Ex.2 An amount of $25,000.00 is deposited in a bank paying an annual interest rate of 3%, compounded monthly. What is the balance after 4 years?

$y = 25000\left(1 + \left(\frac{.03}{12}\right)\right)^{12 \cdot 4} \rightarrow \approx \$28,183.20$

Mar 14-8:11 AM

Ex.3 Suppose an amount of $2,500.00 is deposited in a bank paying an annual interest rate of 5%, compounded quarterly. How many years will it take for the amount in the bank to become $30,000 ?

$$\frac{30000}{2500} = \frac{2500}{2500}\left(1 + \left(\frac{.05}{4}\right)\right)^{4t}$$

$$12 = (1.0125)^{4t}$$

$$\frac{\log(12)}{\log(1.0125)} = 4t$$

$$\frac{\frac{\log(12)}{\log(1.0125)}}{4} = \frac{4t}{4}$$

$t \approx 50$ yrs.

Mar 14-7:18 AM

Half-Life Model:

$$y = a(0.5)^{\frac{t}{k}}$$

y = the amount after t time a = the initial amount

t = time gone by k = the half-life time

Ex.4 An element has a half-life of 30 hours. Write the exponential decay function for a 100-mg sample. Then use the function to find the amount of the element remaining after 7 days. *Round answer to two decimal places.*

168 hrs.

$$y = 100(0.5)^{\frac{168}{30}}$$

2.06 mg

Ex.5 The isotope Dubnium-262 has a half-life of 34 seconds. How long would it take 500 mg. to decay to 1 mg. ?

$$\frac{1}{500} = \frac{500}{500}(0.5)^{\frac{t}{34}}$$

$$.002 = (0.5)^{\frac{t}{34}}$$

$$34 \cdot \left(\frac{\log(.002)}{\log(0.5)}\right) = \frac{t}{34} \cdot 34$$

$t \approx 304.8$ seconds

Mar 14-10:28 AM