D.5 - Exponential/Logarithmic Modeling and Applications

Day 5

Advanced Compound Interest Model:

$$y = P\left(1 + \frac{r}{n}\right)^{nt}$$

y = the amount of money after time has passed

P = the principal (*initial*) amount invested

r = annual rate of interest (*as a decimal*)

t = number of years

n = number of times interest is compounded per year

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Ex.1 An amount of $1,500.00 is deposited in a bank paying an annual interest rate of 4.5%, compounded quarterly. What is the balance after 6 years?

$P = 1500$

$r = .045$

$n = 4$

$t = 6$ $\rightarrow n \cdot t = 24$

$y = 1500\left(1 + \left(\frac{.045}{4}\right)\right)^{24}$

$y = \$1961.99$

Ex.2 An amount of $25,000.00 is deposited in a bank paying an annual interest rate of 3%, compounded monthly. What is the balance after 4 years?

$P = 25000$

$r = .03$

$n = 12$

$t = 4$ $\rightarrow n \cdot t = 48$

$y = 25000\left(1 + \left(\frac{.03}{12}\right)\right)^{48}$

$y = \$28,183.20$

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Ex.3 Suppose an amount of $2,500.00 is deposited in a bank paying an annual interest rate of 5%, compounded quarterly. How many years will it take for the amount in the bank to become $30,000 ?

$P = 2500$

$r = .05$

$n = 4$

$t = ?$

$y = 30000$

$$\frac{30000}{2500} = \frac{2500}{2500}\left(1+\left(\frac{.05}{4}\right)\right)^{4t}$$

$$12 = (1.0125)^{4t}$$

$$\frac{\log(12)}{\log(1.0125)} = 4t$$

$$\frac{200.032412914}{4} = \frac{4t}{4}$$

$t = 50.0$ yrs

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Half-Life Model: $y = a(0.5)^{\frac{t}{k}}$

y = the amount after t time a = the initial amount

t = time gone by k = the half-life time

Ex.4 An element has a half-life of 30 hours. Write the exponential decay function for a 100-mg sample. Then use the function to find the amount of the element remaining after 7 days. *Round answer to two decimal places.*

$k = 30$ hrs.

$a = 100$ mg

$t = 168$ hrs.

$y = 100(0.5)^{\left(\frac{168}{30}\right)}$

$y \approx 2.06$ mg

Ex.5 The isotope Dubnium-262 has a half-life of 34 seconds. How long would it take 500 mg. to decay to 1 mg. ?

$K = 34$ s.

$t = ?$

$a = 500$

$y = 1$

$$\frac{1}{500} = \frac{500}{500}(0.5)^{\left(\frac{t}{34}\right)}$$

$$.002 = (0.5)^{\left(\frac{t}{34}\right)}$$

$$\frac{\log(.002)}{\log(0.5)} = \frac{t}{34}$$

$$34 \cdot 8.96578\ldots = \frac{t}{34} \cdot 34$$

304.8 seconds $= t$

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