Review Unit 2

Name: KEY Date: ____________ Per: ____

UNIT 2 - Review PART 1: 2.1 – Graphing Exponential/Logarithmic Functions

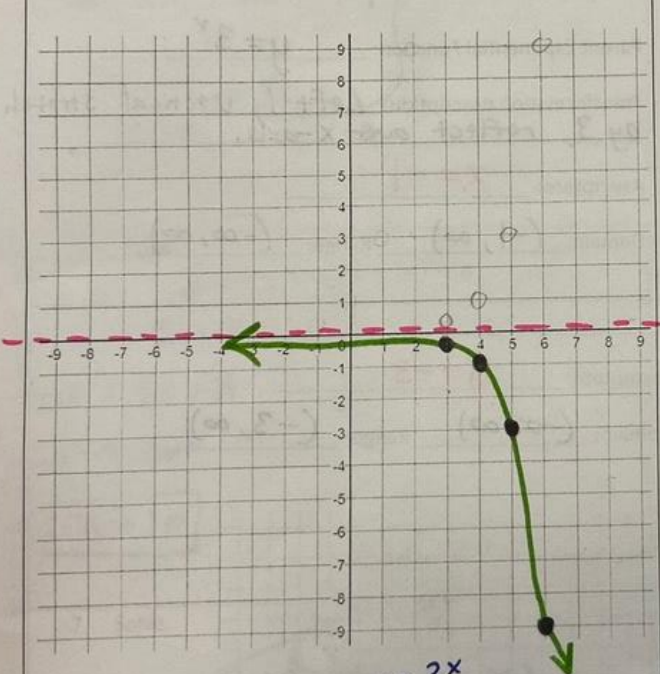
Fill in the blanks using terms and phrases from the word bank below:

domain	asymptote	vertical	horizontal	inverse	reflection	x =	y =

1. Exponential Functions have horizontal asymptotes. These asymptotes have equations that begin with y = .
2. Logarithmic Functions have vertical asymptotes. These asymptotes have equations that begin with X = .
3. I can find the points of a logarithmic function by taking the inverse of the exponential function points.
4. A line in which a graph of a function approaches but never intersects is called an asymptote.

Graph each and fill in all blanks.

5. $f(x) = -(3)^{x-4}$



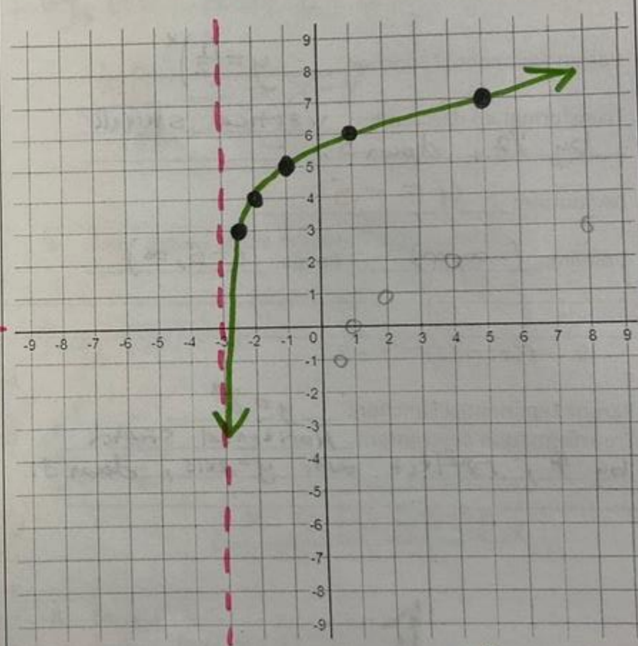
Parent Exponential Function: $y = 3^x$

Transformation description: Right 4, reflect over x-axis.

Asymptote: $y = 0$

Domain: $(-\infty, \infty)$ Range: $(-\infty, 0)$

6. $f(x) = log_2(x + 3) + 4$



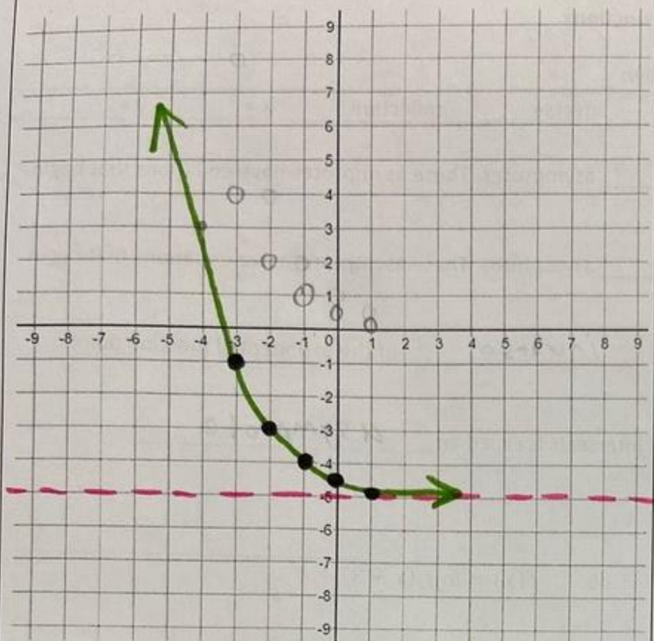
Parent Exponential Function: $y = 2^x$

Transformation description: Left 3, up 4

Asymptote: $X = -3$

Domain: $(-3, \infty)$ Range: $(-\infty, \infty)$

7. $f(x) = \frac{1}{2}\left(\frac{1}{2}\right)^x - 5$



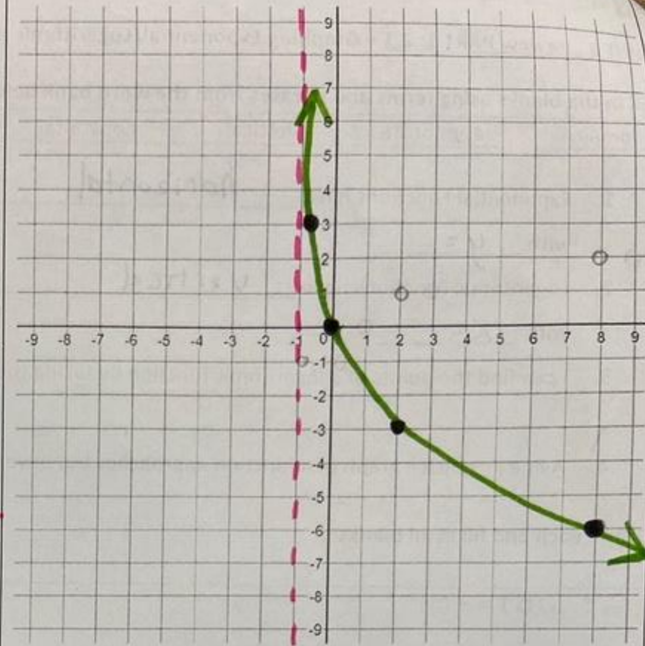
Parent Exponential Function: $y = \left(\frac{1}{2}\right)^x$

Transformation description: vertical shrink by 1/2, down 5.

Asymptote: $y = -5$

Domain: $(-\infty, \infty)$ Range: $(-5, \infty)$

8. $f(x) = -3log_3(x+1)$



Parent Exponential Function: $y = 3^x$

Transformation description: Left 1, vertical stretch by 3, reflect ~~ove~~ over x-axis.

Asymptote: $x = -1$

Domain: $(-1, \infty)$ Range: $(-\infty, \infty)$

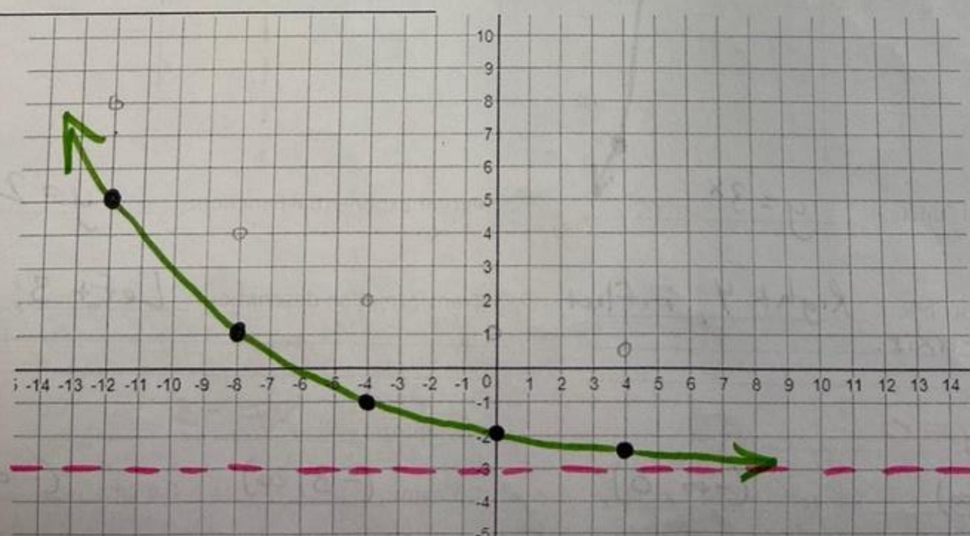
9. $f(x) = (2)^{-.25x} - 3$

Parent Exponential Function: $y = 2^x$

Transformation description: Horizontal stretch by 4, reflect over y-axis, down 3.

Asymptote: $y = -3$

Domain: $(-\infty, \infty)$ Range: $(-3, \infty)$



UNIT 2 - Review PART 2 --- 2.2 – Solving Exponential and Logarithmic Equations –

1. Solve. Answer must be exact, no decimal. $$9^{x-5} = 243^{x-3}$$ $3^{2(x-5)} = 3^{5(x-3)}$ $2x-10 = 5x-15$ $5 = 3x$ $\boxed{x = 5/3}$	2. Solve. *Round to the nearest ten-thousandth.* $$e^{x+3} = 93$$ $\log_e(93) = x+3$ $\ln(93) = x+3$ $-3 \quad -3$ $\boxed{x = 1.5326}$
3. Solve. *Round to the nearest ten-thousandth* $$10^{3x} - 4 = 68$$ $10^{3x} = 72$ $\boxed{x = 0.6191}$ $\log_{10}(72) = 3x$ $\frac{\ln(72)}{\ln(10)} = 3x$	4. Solve. *Round to the nearest ten-thousandth* $$3(5)^{x-4} + 5 = 47$$ $(5)^{x-4} = 14$ $\boxed{x = 5.6397}$ $\log_5(14) = x-4$ $\frac{\ln(14)}{\ln(5)} = x-4$

5. Solve. $$\log_2(5x+14) = 6$$ $2^6 = 5x+14$ $64 = 5x+14$ $50 = 5x$ $\boxed{x=10}$	6. Solve. *Round to the nearest ten-thousandth.* $$\ln(2x) = 3$$ $\log_e(2x) = 3$ $\frac{e^3}{2} = \frac{2x}{2}$ $\boxed{x = 10.0428}$
7. Solve. Power Rule! $$3\log_7 2 - \log_7(x+3) = \log_7 2$$ $\log_7 \frac{2^3}{(x+3)} = \log_7 2$ $\frac{8}{(x+3)} = \frac{2}{1}$ $2x+6 = 8$, $2x = 2$, $\boxed{x=1}$	8. Solve. $$\log_3(x+4) - \log_3(2) = 2$$ $\log_3 \frac{(x+4)}{2} = 2$ $2 \cdot 3^2 = \frac{(x+4)}{2} \cdot 2$ $18 = x+4$ $\boxed{14 = x}$

9. Solve. Product Rule! $\log x + \log (x-15) = 2$ $\log x^2-15x = 2$ $10^2 = x^2-15x$ $0 = x^2-15x-100$ $0 = (x-20)(x+5)$ $\boxed{x=20}$ $\cancel{x=-5}$	10. Solve. Power Rule! $2\log_5 x - \log_5 9 = 2$ $\log_5 \frac{x^2}{9} = 2$ $5^2 = \frac{x^2}{9}$ $9 \cdot 25 = \frac{x^2}{9} \cdot 9$ $225 = x^2$ $\sqrt{225} = \sqrt{x^2}$ $\pm 15 = x$ $\boxed{x=15}$ $\cancel{x=-15}$
11. Solve. $\log_3 (x-4) + \log_3 (x+1) = \log_3 (x+8)$ $\log_3 x^2-3x-4 = \log_3 (x+8)$ $x^2-3x-4 = x+8$ $x^2-4x-12 = 0$ $(x-6)(x+2) = 0$ $\boxed{x=6}$ $\cancel{x=-2}$	12. Solve. $\ln (x+3) - (\ln 6 + \ln x) = \ln \left(\frac{1}{4}\right)$ $\ln (x+3) - \ln 6x = \ln \frac{1}{4}$ $\ln \frac{(x+3)}{6x} = \ln \frac{1}{4}$ $\frac{x+3}{6x} = \frac{1}{4}$ $6x = 4x+12$ $2x = 12$, $\boxed{x=6}$
13. Solve. $\log_2(x-3) + \log_2(x) - \log_2(x+2) = 2$ $\log_2 x^2-3x - \log_2 (x+2) = 2$ $\log_2 \frac{x^2-3x}{x+2} = 2$ $2^2 = \frac{x^2-3x}{x+2}$ $\frac{4}{1} = \frac{x^2-3x}{x+2}$ $x^2-3x = 4x+8$ $x^2-7x-8 = 0$ $(x-8)(x+1) = 0$ $\boxed{x=8}$ $\cancel{x=-1}$	14. Solve. $2 \log_2 (4x) - \log_2 6 = 3$ $\log_2 \frac{16x^2}{6} = 3$ $2^3 = \frac{16x^2}{6}$ $8 = \frac{16x^2}{6}$ $48 = 16x^2$ $3 = x^2$ $\pm\sqrt{3} = \sqrt{x^2}$ $\boxed{x=\sqrt{3}}$ $\cancel{x=-\sqrt{3}}$

UNIT 2 - Review PART 3 --- 2.3 – **Exponential and Logarithmic Apps. and Modeling**

Formulas:

Continuous Compounded Interest:	Half-Life:	Newton's Law of Cooling:
$A = Pe^{rt}$	$A = A_o(0.5)^{\frac{t}{k}}$	$T = C + (T_o - C)\cdot e^{kt}$

Basic Growth/Decay Expo. Formula:	"n" Compounding Periods of Interest:
$A = A_o(1 \pm r)^t$	$A = P\left(1 + \left(\frac{r}{n}\right)\right)^{nt}$

2. An amount of $7,200 is deposited in a bank paying an interest rate of 4%, compounded monthly. What is the balance after 5 years? (*Round to the nearest hundredth*)

$$A = 7200\left(1+\left(\frac{.04}{12}\right)\right)^{(12\cdot 5)}$$

$$\boxed{A = \$\,8791.18}$$

3. The half-life of Cesium-137 is 30 years. It is highly radioactive. Determine the initial amount in grams if there was 6 grams after 68 years rounded to the nearest gram *(do not count leap year days).*

$$\frac{6}{(0.5)^{\left(\frac{68}{30}\right)}} = \frac{A_o\cdot(0.5)^{\left(\frac{68}{30}\right)}}{(0.5)^{\left(\frac{68}{30}\right)}}$$

$$A_o = 28.87\ldots \rightarrow \boxed{29 \text{ grams}}$$

4. A population of largemouth bass in a lake has been steadily decreasing by a rate of 21 % per year for many years now since the year 2000. It was estimated that this year (2018) there is currently only about 400 largemouth bass alive in the lake. What was the amount in 2000?

$t = 18$

$$\frac{400}{(1-.21)^{18}} = \frac{A_o\cdot(1-.21)^{18}}{(1-.21)^{18}}$$

$$A_o = 27846.611$$

$$\boxed{A_o = 27847 \text{ bass}}$$

5. The growth model $A = 4.1e^{0.01x}$ describes New Zealand's population, A in millions, x years after 2006. In what year will New Zealand's population be at 6.56 million? (*Round to the nearest year*)

$$\frac{6.56}{4.1} = \frac{4.1e^{0.01x}}{4.1}$$

$$1.6 = e^{0.01x}$$

$$\log_e(1.6) = 0.01x$$

$$\frac{\ln(1.6)}{0.01} = \frac{0.01x}{0.01}$$

$$x = 47.00036\ldots$$

Yr. 2006
+ 47 yrs

$\boxed{\text{Yr. } 2053}$

6. An amount of \$800 is deposited in a bank and for 5 years compounded continuously. At what rate should it be compounded to reach an amount of \$2,400? (*Round to the nearest tenth of a percent*)

$$A = P \cdot e^{rt}$$

$$\frac{2400}{800} = \frac{800}{800} \cdot e^{5r}$$

$$3 = e^{5r}$$

$$\log_e(3) = 5r$$

$$\frac{\ln(3)}{5} = \frac{5r}{5}$$

$$r = 0.219722..$$

$$\times 100$$

$$21.97 \rightarrow \boxed{21.9\%} \text{ OR } \boxed{22\%}$$

7. An amount of \$25,000 is deposited in a bank paying an interest rate of 3.5%, compounded semi-annually. How many years will it take for that amount to become \$ 35,000? (*Round to the nearest tenth*)

$$\frac{35000}{25000} = \frac{25000}{25000}\left(1 + \left(\frac{.035}{2}\right)\right)^{(2t)}$$

$$1.4 = (1.0175)^{2t}$$

$$\log_{1.0175}(1.4) = 2t$$

$$\frac{\ln(1.4)}{\ln(1.0175)} \Big/ 2 = \frac{2t}{2}$$

$$\boxed{t = 9.7 \text{ yrs.}}$$

8. The half-life of a certain radioactive substance is 50 minutes. There are 300 grams in the initial sample. How long (in hours, minutes, seconds) exactly until there will be 5 grams remaining?

$K = 50$ min.

$$5 = 300(0.5)^{\frac{t}{50}}$$

$$\frac{5}{300} = (0.5)^{t/50}$$

$$\log_{0.5}\left(\frac{5}{300}\right) = \frac{t}{50}$$

$$\frac{\ln(5/300)}{\ln(0.5)} = \frac{t}{50}$$

$$t = 295.3445298.. \text{ Minutes}$$

$$\boxed{\text{4 hours, 55 min, 21 secs}}$$

9. The half-life of a certain radioactive substance is 36 days. There are 24 grams in the initial sample. How long exactly until there will be 1 gram remaining? (Round to the nearest day)

$K = 36$ days

$$1 = 24(0.5)^{\frac{t}{36}}$$

$$\frac{1}{24} = (0.5)^{t/36}$$

$$\log_{0.5}\left(\frac{1}{24}\right) = \frac{t}{36}$$

$$\frac{\ln\left(\frac{1}{24}\right)}{\ln(0.5)} = \frac{t}{36}$$

$$t = 165.05865... \text{ days}$$

$$\boxed{\text{165 days}}$$

10. A population of bacteria cells can double every 3 hours. If the initial count is 25, how many days, hours, minutes, seconds will it take for the count to reach 10,000,000 ?

$K = 3$ hrs.

$$10\,000\,000 = 25(2)^{\frac{t}{3}}$$

$$400\,000 = (2)^{t/3}$$

$$\log_2(400000) = \frac{t}{3}$$

$$\frac{\ln(400000)}{\ln(2)} = \frac{t}{3}$$

$t =$ hours

$55.82892142\ldots$

$\boxed{\text{2 days, 7 hrs. 49 min 44 secs}}$

11. The city of Smallville, IL has had an increasing population since 2011 at a rate of about 1.8 %. The population in 2011 was about 7200 people. If this trend continues, what will be the population in 2024?

$t = 13$

$$A = 7200(1 + .018)^{13}$$

$$A = 9079.32$$

$\boxed{\text{9079 people in 2024}}$

12. Suppose that a corpse was discovered in a motel room at 7:30 AM and the dead body's temperature was 71° F . The temperature of the room was kept constant at 60° F. One hour later the temperature of the corpse was taken again, and found to be 67° F. Find the exact time of death. *(Hint: the temperature of a living human body is normally 98.6° F)*

$K = ?$

$$67 = 60 + (71 - 60) \cdot e^{60K}$$

$$7 = 11 \cdot e^{60K}$$

$$\frac{7}{11} = e^{60K}$$

$$\ln\left(\frac{7}{11}\right) = 60K$$

$$\boxed{K = -0.0075}$$

T.O.D = ?

$$71 = 60 + (98.6 - 60) \cdot e^{Kt}$$

$$11 = 38.6 \cdot e^{Kt}$$

$$\frac{11}{38.6} = e^{Kt}$$

$$\ln\left(\frac{11}{38.6}\right) = -0.0075t$$

$$t = 167.38$$

$t = 167$ minutes

7:30 AM
− 2 hrs.
− 47 minutes

T.O.D. :

$\boxed{\text{4:43 AM}}$