14-3 (D.7) - Trigonometric Applications and Modeling

take note

Key Concept Cosecant, Secant, and Cotangent Functions

The **cosecant** (csc), **secant** (sec), and **cotangent** (cot) functions are defined using reciprocals. Their domains do not include the real numbers θ that make the denominator zero.

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

($\cot\theta = 0$ at odd multiples of $\frac{\pi}{2}$, where $\tan\theta$ is undefined.)

take note

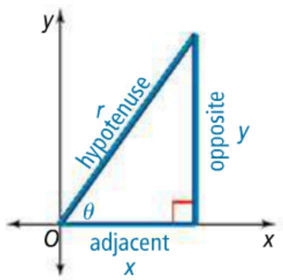
Key Concept Trigonometric Ratios for a Right Triangle

If θ is an acute angle of a right triangle, x is the length of the adjacent leg (ADJ), y is the length of the opposite leg (OPP), and r is the length of the hypotenuse (HYP), then the trigonometric ratios of θ are as follows.

$$\sin\theta = \frac{y}{r} = \frac{\text{OPP}}{\text{HYP}} \qquad \csc\theta = \frac{r}{y} = \frac{\text{HYP}}{\text{OPP}}$$

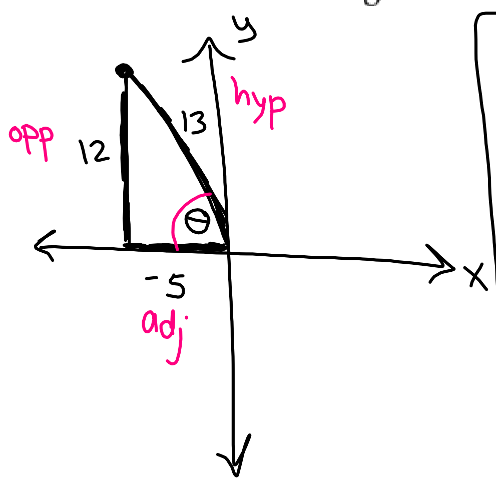
$$\cos\theta = \frac{x}{r} = \frac{\text{ADJ}}{\text{HYP}} \qquad \sec\theta = \frac{r}{x} = \frac{\text{HYP}}{\text{ADJ}}$$

$$\tan\theta = \frac{y}{x} = \frac{\text{OPP}}{\text{ADJ}} \qquad \cot\theta = \frac{x}{y} = \frac{\text{ADJ}}{\text{OPP}}$$



May 4-7:27 AM

1. For a standard-position angle determined by the point $(-5, 12)$, what are the values of the six trigonometric functions?



$\sin\theta = \frac{12}{13}$ $\qquad \csc\theta = \frac{13}{12}$

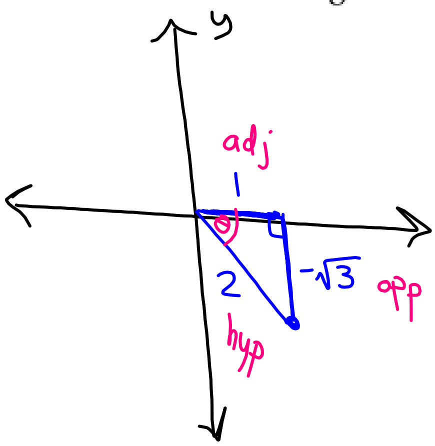
$\cos\theta = -\frac{5}{13}$ $\qquad \sec\theta = -\frac{13}{5}$

$\tan\theta = -\frac{12}{5}$ $\qquad \cot\theta = -\frac{5}{12}$

May 4-7:32 AM

2. For a standard-position angle determined by the point $(1, -\sqrt{3})$ what are the values of the six trigonometric functions?

x y



$\boxed{\sin\theta = -\frac{\sqrt{3}}{2}}$

$\csc\theta = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{\cos\theta = \frac{1}{2}}$

$\boxed{\csc\theta = -\frac{2\sqrt{3}}{3}}$

$\boxed{\sec\theta = 2}$

$\boxed{\tan\theta = -\sqrt{3}}$

$\cot\theta = \frac{1}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \boxed{\cot\theta = -\frac{\sqrt{3}}{3}}$

Apr 4-8:17 AM

More advanced elevation/depression problem...

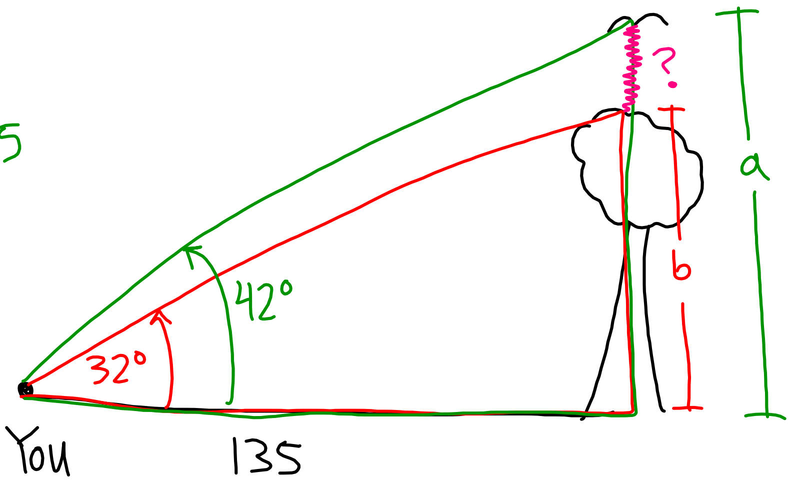
A bird is flying above a tree. You are standing 135 feet away from the tree. The angle of elevation to the top of the tree is 32°, and the angle of elevation to the bird is 42°. What is the distance from the bird to the top of the tree?

$\tan(42) = \frac{a}{135}$

$a = 121.6$

$\tan(32) = \frac{b}{135}$

$b = 84.4$



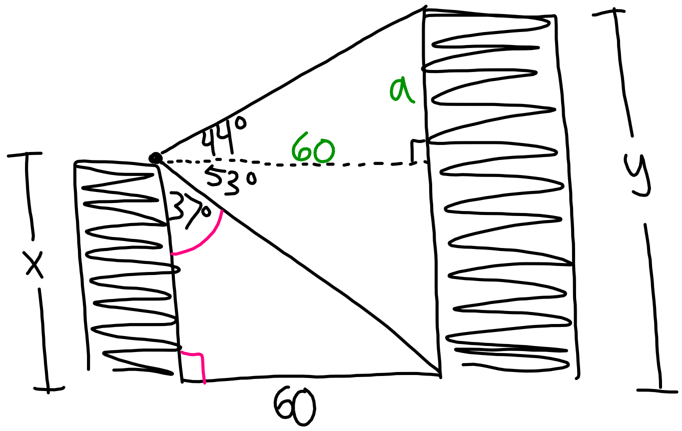
$? = 121.6 - 84.4$

$\boxed{? = 37.2 \text{ ft.}}$

May 4-7:39 AM

More advanced elevation/depression problem...

Two students want to determine the heights of two buildings. They stand on the roof of the shorter building. The students use a clinometer to measure the angle of elevation of the top of the taller building. The angle is 44. From the same position, the students measure the angle of depression of the base of the taller building. The angle is 53. The students then measure the horizontal distance between the two buildings. The distance is 60 ft. How tall is each building?



$\tan(44) = \frac{a}{60}$

$a = 57.9$

$+\ 79.6$

$\boxed{y = 137.5 \text{ ft}}$

Apr 4-12:45 PM